Spectral Analysis of Short Time Signals

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The very old problem of extracting frequencies from time signals is addressed in the case of signals that are very short as compared to their intrinsic time scales. The solution of the problem is not only important to the classic signal processing but also helps to disqualify several common formulations of the quantum mechanical time energy uncertainty principle. Our purpose is to identify the source of the difficulty with a spectral decomposition of short signals and introduce a tool to perform such a decomposition when possible.

The goal of many scientific efforts is to predict future evolution of physical systems on the basis of their known past behavior. One example of the very limited success of such an activity is weather forecasts. Even if the history of all important parameters like temperature, pressure, humidity, wind velocity, etc., is known for many years back at almost every point on Earth, the reasonably accurate prediction of the coming weather conditions can be made only for several days. One may argue that Earth's atmosphere is an especially tough system to consider due to its intrinsic instabilities: even a tiny perturbation of air in one place can lead to huge changes of weather on a distant continent. In this work we will not be able to deal with such unstable systems either.

The other extreme is represented by very stable systems such as celestial objects. Observations of the Moon and the Sun, that took centuries, allowed ancient astronomers to predict accurately the Moon's phases, risings, settings, and eclipses for coming millennia without any knowledge of gravitational forces or Kepler's laws. Such precision was possible because the observations of the system had been made over a much longer period than the system's characteristic time scales: days, (sidereal) months, and years.

Would the same quality predictions be possible if we observed the Moon just for 15 minutes, i.e., for a time much shorter than the shortest characteristic time scale? In this work we show a practical way of achieving exact predictions of a future based on a very short history of a system. Our predictions will be limited only to the quantities evolution of which can be well described by finite Fourier series. This restriction is crucial. Still, there are many important quantities that fall into this category.

Similar problems, but for signals known over a long time, are usually treated by a discrete Fourier transform method (DFT). That method *assumes* a grid of equidistant frequencies and finds corresponding amplitudes only. The spacing of the assumed frequencies is proportional to the inverse of the time span of the signal and thus the method is useless in cases where this time is very short and the distances between frequencies are small.

Harmonic inversion, on the other hand, performs the more challenging task of extracting both amplitudes and frequencies. The method can be applied to any signal of classical or quantum origin as long as it has the form of finite Fourier series (the number of Fourier terms need not be known in advance). When applied to quantum systems it gives important insight to the problem of validity of several formulations of the time-energy uncertainty relations. Interested readers can see [1] for derivation and details of harmonic inversion.

Coming back to the problem of determining the future positions of the Moon in the sky just from a short observation we see that it is possible under one condition: the observation has to be very accurate. The major obstacle in achieving required accuracy would be refraction of the incoming moonbeams in the Earth's atmosphere, which results in a significant shift of the apparent Moon's position with respect to the actual position.

Figure 1 shows a practical example of extracting accurate frequencies and amplitudes from a signal that is available only over a very short period of time. The only requirement is that the signal itself must be precise. The required precision is not achievable in real-world experiments where observed signals are short and involve many frequencies. The situation

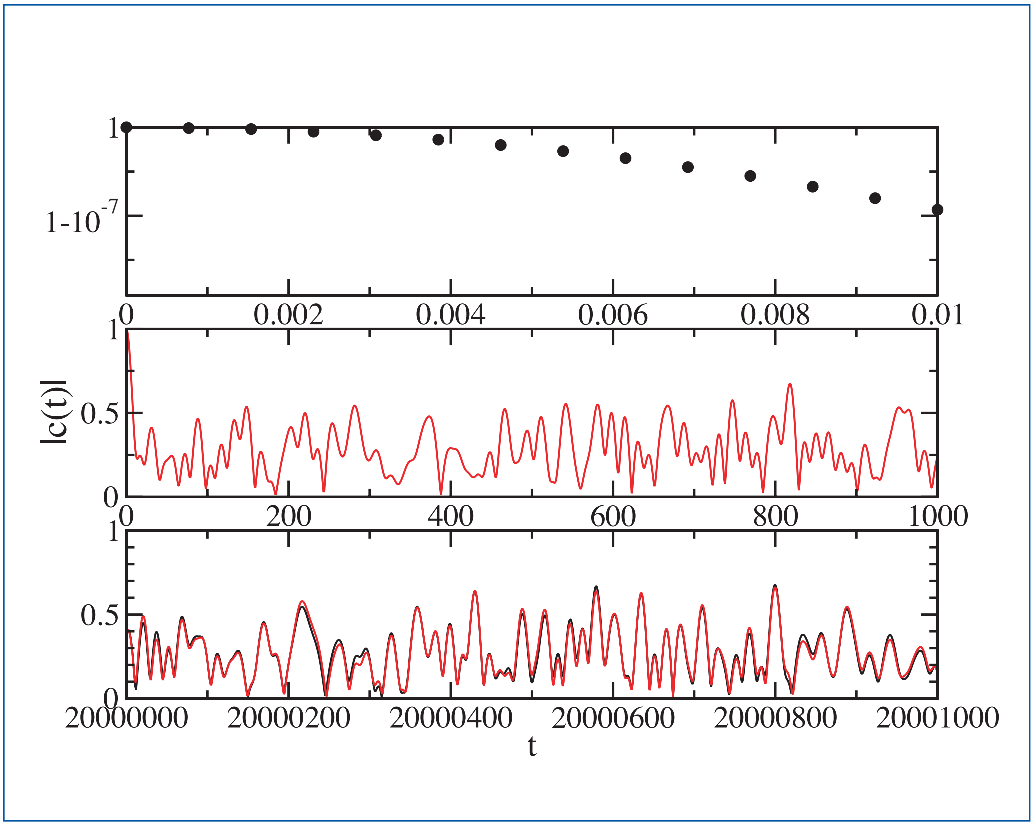

Figure 1—
As an example, in the upper plot, the harmonic inversion method was applied to a signal with 10 frequencies randomly drawn from the interval (0.5,1.0), sampled at 14 points over the time 0.01. Based on the points of the upper plot, the middle plot shows the signal reconstructed (red line) with the help of harmonic inversion. Exact (black line) and reconstructed (red line) signals are indistinguishable. In fact, the two lines start to differ by 1% for times longer than 1,000,000. The discrepancies, that are due to roundoff errors (noise) of the initial points, are visible in the bottom plot. As a curiosity, the DFT algorithm applied to this signal would give just one frequency equal to zero, so that the prediction would be a horizontal line at 1.

is somewhat better in numerical simulations where one has more control over generated data. The method equips us also with a powerful argument against some interpretations of time-energy uncertainty relations in quantum mechanics.

[1] Z.P. Karkuszewski, arXiv:quant-ph/0412073.